Aerodynamic Force and Line of Action Model

This planar whole-vehicle model estimates aerodynamic force, moment and line of action for validation. O is the front-wheel-axis datum, G is the whole-vehicle center of mass, and P is the point on the line closest to G. Use road-aligned axes: +x horizontally forward, +z vertically down and +y to the right. Positive y-moment pitches the nose upward. O translates with the car; these axes do not pitch with it. Time arguments are suppressed after the vector definitions.

Inputs and parameters

Inputs are synchronized, processed quantities. Express CG position and gravity-corrected inertial acceleration in the road-aligned frame. Wheel and crankshaft rates are signed about +y relative to the chassis pitch frame. Forward-rolling wheels therefore have negative absolute spin. Inertias and nominal contact geometry are calibrated parameters.

$$\mathbf{u}_A(t) = \begin{bmatrix} N_F(t) \\ N_R(t) \\ a_{G,x}(t) \\ a_{G,z}(t) \\ \omega_y(t) \\ \Omega_{\mathrm{FL}}(t) \\ \Omega_{\mathrm{FR}}(t) \\ \Omega_{\mathrm{RL}}(t) \\ \Omega_{\mathrm{RR}}(t) \\ \Omega_e(t) \\ x_{G/O}(t) \\ z_{G/O}(t) \end{bmatrix}$$

$$\mathbf{r}_{G/O}(t) = \begin{bmatrix} x_{G/O}(t) \\ z_{G/O}(t) \end{bmatrix}$$

$$I_{w,\mathrm{FL}} = I_{w,\mathrm{FR}} = I_{w,\mathrm{RL}} = I_{w,\mathrm{RR}} = I_w$$

$$\mathbf{p}_A = \begin{bmatrix} m \\ g \\ I_{G,\mathrm{yy}}^{\mathrm{locked}} \\ I_w \\ I_e \\ D_0 \\ K \\ r_F \\ x_{F/O} \\ z_{F/O} \\ x_{R/O} \\ z_{R/O} \end{bmatrix}$$

The locked inertia is the whole-vehicle pitch inertia with rotor spin locked relative to the chassis, held constant in this approximation. The four wheels each have spin inertia Iw. Ie is the crankshaft and rigidly co-rotating assembly inertia; other drivetrain rotors require the treatment on the validation page. rF is the front rolling radius. D_0 and K are empirical force-relation parameters. Use SI units, radians and seconds.

Aerodynamic outputs and force balances

L is positive downforce, D is positive drag magnitude, and $M_{(}A,G_{)}$ is the signed aerodynamic moment about G. Front drive thrust is zero. T is the signed net rear longitudinal tire-road force, rather than crankshaft torque divided by wheel radius.

$$\mathbf{y}_A(t) = \begin{bmatrix} L(t) \\ D(t) \\ M_{A,G}(t) \end{bmatrix}$$

$$\mathbf{y}_A(t) = f_A(\mathbf{u}_A(t), \dot{u}_A(t); \mathbf{p}_A)$$

$$\mathbf{F}_A = \begin{bmatrix} -D \\ L \end{bmatrix}$$

$$\mathbf{F}_g = \begin{bmatrix} 0 \\ mg \end{bmatrix}, \quad \mathbf{a}_G = \begin{bmatrix} a_{G,x} \\ a_{G,z} \end{bmatrix}$$

$$\mathbf{F}_{N,F} = \begin{bmatrix} 0 \\ -N_F \end{bmatrix}, \quad \mathbf{F}_{N,R} = \begin{bmatrix} 0 \\ -N_R \end{bmatrix}$$

$$\mathbf{F}_{X,F} = \begin{bmatrix} X_F \\ 0 \end{bmatrix}, \quad \mathbf{F}_{T,R} = \begin{bmatrix} T \\ 0 \end{bmatrix}$$

$$N_F = N_{\mathrm{FL}} + N_{\mathrm{FR}}, \quad N_R = N_{\mathrm{RL}} + N_{\mathrm{RR}}$$

$$T_{\mathrm{drive},F} = 0$$

$$\mathbf{F}_A + \mathbf{F}_g + \mathbf{F}_{N,F} + \mathbf{F}_{N,R} + \mathbf{F}_{X,F} + \mathbf{F}_{T,R} = m\mathbf{a}_G$$

$$mg + L - N_F - N_R = ma_{G,z}$$

$$L = N_F + N_R - mg + ma_{G,z}$$

$$D = D_0 + KL^2$$

$$X_F + T - D = ma_{G,x}$$

$$T = ma_{G,x} + D - X_F$$

NF and NR denote tire-road normal forces; suspension sensor readings require conversion to those forces. X_F is the signed front road force and is usually negative during front-wheel spin-up. The empirical drag relation applies only within its calibration range.

Rotational and contact geometry

Each force location is obtained by subtracting the time-varying CG position from its contact-point position in the road-aligned frame. F and R denote front and rear contact patches. Fixed contact coordinates are a nominal-geometry approximation.

$$\alpha_y = \dot{\omega}_y$$

$$\dot{\Omega}_i = \frac{d\Omega_i}{dt}, \quad \dot{\Omega}_e = \frac{d\Omega_e}{dt}$$

$$\mathbf{r}_{F/O} = \begin{bmatrix} x_{F/O} \\ z_{F/O} \end{bmatrix}, \quad \mathbf{r}_{R/O} = \begin{bmatrix} x_{R/O} \\ z_{R/O} \end{bmatrix}$$

$$\mathbf{r}_{F/G} = \mathbf{r}_{F/O} - \mathbf{r}_{G/O}$$

$$\mathbf{r}_{R/G} = \mathbf{r}_{R/O} - \mathbf{r}_{G/O}$$

$$x_{F/G} = x_{F/O} - x_{G/O}, \quad x_{R/G} = x_{R/O} - x_{G/O}$$

$$z_{F/G} = z_{F/O} - z_{G/O}, \quad z_{R/G} = z_{R/O} - z_{G/O}$$

$$M_{i,G,y} = z_{i/G} F_{i,x} - x_{i/G} F_{i,z}$$

Front wheel spin and longitudinal moment

For ideal free-rolling front wheels with negligible bearing and rolling-resistance moments and no front braking, signed spin acceleration estimates the backward front road force. Wheel-carrier pitch rate is approximated by chassis pitch rate. Both tire contact patches lie on the same flat, level ground plane.

$$\dot{\Omega}_{\mathrm{FL,abs}} = \alpha_y + \dot{\Omega}_{\mathrm{FL}}, \quad \dot{\Omega}_{\mathrm{FR,abs}} = \alpha_y + \dot{\Omega}_{\mathrm{FR}}$$

$$X_F \approx \frac{I_w}{r_F} (\dot{\Omega}_{\mathrm{FL,abs}} + \dot{\Omega}_{\mathrm{FR,abs}})$$

$$z_{F/G} = z_{R/G} = h_G$$

$$h_G = z_{\mathrm{ground}/O} - z_{G/O}$$

$$M_{\mathrm{front},G} = z_{F/G} X_F, \quad M_{\mathrm{thrust},G} = z_{R/G} T$$

$$M_{\mathrm{long},G} = z_{F/G} X_F + z_{R/G} T$$

$$M_{\mathrm{long},G} = h_G (X_F + T) = h_G (m a_{G,x} + D)$$

The spin-up force changes the rear thrust inferred from longitudinal acceleration. At equal contact heights, its contribution cancels from the combined longitudinal moment when the force balance is substituted. The front wheel spin inertia remains in the angular-momentum balance.

Moment balance about G

Gravity passes through G and contributes zero moment. The external aerodynamic, normal, and longitudinal tire-force moments equal the rate of change of vehicle and rotor angular momentum.

$$M_{\text{weight},G} = 0, \quad M_{\text{normal},G} = x_{F/G}N_F + x_{R/G}N_R$$

$$H_{G,y} = I_{G,\text{yy}}^{\text{locked}}\omega_y + I_w(\Omega_{\text{FL}} + \Omega_{\text{FR}} + \Omega_{\text{RL}} + \Omega_{\text{RR}}) + I_e\Omega_e$$

$$M_{\text{dynamic},G} = \frac{dH_{G,y}}{dt}$$

$$M_{\text{pitch},G} = I_{G,\text{yy}}^{\text{locked}}\alpha_y$$

$$M_{\text{wheels},G} = I_w\sum_{i\in W}\dot{\Omega}_i\,, \quad M_{\text{engine},G} = I_e\dot{\Omega}_e$$

$$M_{\text{dynamic},G} = M_{\text{pitch},G} + M_{\text{wheels},G} + M_{\text{engine},G}$$

$$M_{\text{dynamic},G} = I_{G,\text{yy}}^{\text{locked}}\alpha_y + I_w(\dot{\Omega}_{\text{FL}} + \dot{\Omega}_{\text{FR}} + \dot{\Omega}_{\text{RL}} + \dot{\Omega}_{\text{RR}}) + I_e\dot{\Omega}_e$$

$$M_{A,G} + M_{\text{normal},G} + M_{\text{long},G} = M_{\text{dynamic},G}$$

$$M_{A,G} = M_{\text{dynamic},G} - M_{\text{normal},G} - h_G(ma_{G,x} + D)$$

$$M_{A,G} = I_{G,\text{yy}}^{\text{locked}}\alpha_y + I_w\sum_{i\in W}\dot{\Omega}_i + I_e\dot{\Omega}_e$$
$$-x_{F/G}N_F - x_{R/G}N_R - h_G(ma_{G,x} + D)$$

The rotor rates are relative to the vehicle pitch frame; the locked inertia accounts for their shared pitch rotation. The common-height reduction relies on a flat ground plane. Motion of suspension components relative to the vehicle is omitted from the angular-momentum approximation except through the supplied CG position. W indexes the four wheels: FL, FR, RL and RR. Wheel and engine spin terms are angular-momentum rates, rather than additional external moments on the whole car.

All spin rates must use the stated sign convention. The inferred aerodynamic moment also contains the effect of any omitted external couple, such as a tire rolling-resistance moment. Bound or calibrate those effects before interpreting the residual as aerodynamic moment.

Aerodynamic line of action

The resultant magnitude is R. The unit direction vector points along the aerodynamic force; the chosen unit normal makes positive signed distance correspond to positive aerodynamic moment.

$$R = \sqrt{D^2 + L^2}, \quad R > 0$$

$$\hat{\mathbf{e}}_A = \frac{1}{R}\begin{bmatrix} -D \\ L \end{bmatrix}, \quad \hat{\mathbf{n}}_A = \frac{1}{R}\begin{bmatrix} -L \\ -D \end{bmatrix}$$

$$\hat{\mathbf{e}}_A \cdot \hat{\mathbf{n}}_A = 0$$

$$\mathbf{r}_{P/G} = d\hat{\mathbf{n}}_A$$

$$M_{A,G} = z_{P/G}(-D) - x_{P/G}L = dR$$

$$d = \frac{M_{A,G}}{R}, \quad |d| = \frac{|M_{A,G}|}{R}$$

$$\mathbf{r}_{P/G} = \frac{M_{A,G}}{D^2 + L^2}\begin{bmatrix} -L \\ -D \end{bmatrix}$$

$$\mathbf{r}_{P/O} = \mathbf{r}_{G/O} + \mathbf{r}_{P/G}$$

$$\mathbf{r}_{\ell/G}(s) = \mathbf{r}_{P/G} + s\hat{\mathbf{e}}_A, \quad s \in \mathbb{R}$$

$$\mathbf{r}_{\ell/O}(s) = \mathbf{r}_{G/O} + \mathbf{r}_{P/G} + s\hat{\mathbf{e}}_A$$

$$Lx_{\ell/G} + Dz_{\ell/G} + M_{A,G} = 0$$

$$\mathcal{L}_A = (\mathbf{r}_{P/O}, \hat{\mathbf{e}}_A)$$

P is the closest point on the line to G. The signed coordinate s selects any other point along it. The ordered pair defining the line contains a point-position vector and a unit direction vector. If R approaches zero, the line location becomes undefined or poorly conditioned.

Ground plane intersection

Set s to the value at which the line reaches ground height. This identifies a ground-plane intersection of the aerodynamic line, rather than a physical aerodynamic application point at the ground.

$$z_{\text{ground}/O} = z_{F/O} = z_{R/O}$$

$$z_{\ell/O}(s_{\text{ground}}) = z_{\text{ground}/O}$$

$$s_{\text{ground}} = \frac{z_{\text{ground}/O} - z_{G/O} - z_{P/G}}{L/R}, \quad L \neq 0$$

$$x_{\text{ground}/O} = x_{G/O} + x_{P/G} - \frac{D}{R}s_{\text{ground}}$$

$$x_{\text{ground}/O} = x_{G/O} - \frac{M_{A,G} + Dh_G}{L}$$

When L = 0, the line is parallel to the ground plane, so it has no unique ground intersection.

Validation scope and checks

This is a baseline for controlled planar validation. Numerical agreement with its own equations establishes algebraic consistency; physical validation requires independent load references and uncertainty estimates. Assume straight travel, level ground and approximately horizontal relative airflow.

Signal and geometry checks

Verify signs, SI units, clock alignment, differentiation and filter delay. Correct off-CG acceleration for sensor location and orientation. Transform suspension geometry to the road frame. Correct wheel encoder rates if carrier pitch differs from chassis pitch. Convert suspension loads to tire-road normals, including unsprung-mass dynamics when material.

Drag closure

D_0 has units N and K has units 1/N. A coefficient drag polar does not supply constant force-level parameters across changing dynamic pressure. Fit and independently check $D = D_0 + K L^2$ over the intended operating range; do not extrapolate a running-speed fit to standstill. Without this validated closure or an independent longitudinal-force reference, drag and the full line are not uniquely measured.

Additional rotating and moving parts

Other significant y-axis drivetrain rotors add signed angular momentum. For a selected rotor set J, excluding wheels already counted, constant signed speed ratios kj allow the engine term to use the angular-momentum coefficient below. It uses signed ratios, not the squared ratios of kinetic-energy reflected inertia.

$$\Omega_j = k_j \Omega_e, \quad I_e^{(H)} = \sum_{j \in J} I_j \, k_j, \quad H_{\mathrm{rot},y} = I_e^{(H)} \Omega_e$$

Replace Ie by this coefficient only for a verified fixed-ratio state; otherwise add each rotor's individual angular-momentum rate. Significant suspension motion requires variable locked inertia and relative-motion terms beyond the baseline approximation. Engine-cycle vibration remains outside scope.

Known load and line checks

Use independently known vertical and pitching loads, then exercise wheel and engine acceleration terms. For this geometry-only check, forces are in N, moments in N m and positions in m. The upper sign corresponds to positive moment.

$$D = 1, \quad L = 10, \quad M_{A,G} = \pm 2, \quad \mathbf{r}_{P/G} = \pm \begin{bmatrix} \frac{-20}{101} \\ \frac{-2}{101} \end{bmatrix}$$

Every point on the recovered line must reproduce the specified moment. For the full flat-ground model, the following equivalent expression cross-checks the ground intersection. Drag cancels here but remains necessary for line direction.

$$x_{\mathrm{ground}/O} = x_{G/O} - \frac{M_{\mathrm{dynamic},G} - M_{\mathrm{normal},G} - h_G m a_{G,x}}{L}$$

Set acceptance tolerances from measurement uncertainty. Propagate force, moment and geometry uncertainty to location; flag near-zero resultant or downforce before reporting a line or ground intersection.

Technical references: MIT rigid-body dynamics; NASA drag equation; NASA induced drag